

**METHOD AND APPARATUS FOR PROTECTING NTRU AGAINST A TIMING
ATTACK**

This application is a non-provisional application claiming priority from United States Provisional Application No. 60/433,015 filed December 13, 2002 and United States Provisional Application No. 60/459,268 filed April 2, 2003.

BACKGROUND OF THE INVENTION

FIELD OF THE INVENTION

[0001] The present invention relates to a method and apparatus for performing public key cryptography.

DESCRIPTION OF THE PRIOR ART

[0002] When communicating over public networks, it is often necessary to secure communications in order to prevent interception or fraud by a third party. Cryptographic schemes often use intractable mathematical problems to ensure security of communications. In private key systems, two correspondents share a secret key prior to initiating communications. They can then employ an encryption algorithm using the secret value to keep their communication private from those who do not know the secret value. However, with such systems it is necessary for the two correspondents to agree on the secret beforehand, which may be as difficult as communicating securely in the first place.

[0003] Public key cryptosystems address the problem of distributing keys by assigning a pair of keys to each user. Each user has a private key and a corresponding public key, which are mathematically related so that it is computationally infeasible to derive the private key from the public key. The public key may be published and therefore made widely available to all users. To encrypt a message for a particular recipient, the sender uses the recipient's public key. Only the recipient knows the corresponding private key and therefore is the only party able to decrypt the message.

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[0004] NTRU is a public key encryption system described in US Patent No, 6,081,597. The NTRU system uses a mathematical structure called a truncated ring of polynomials, which is denoted by R . The NTRU system uses four publicly known system parameters to initially set up the system. These are the degree of polynomials N , two moduli p , q , and the window parameter T . Typically, p is chosen to be 3 or $X+2$, and q is chosen to be a power of 2. The elements of the ring R may be represented as polynomials of a degree less than N . Operations in the ring are performed by polynomial addition and multiplication with the additional identity that $X^N = 1$.

[0005] To generate keys in the NTRU system, each user chooses secret polynomials f and g in the ring R . From the polynomial f , the user computes inverses modulo p and q which are denoted as f_q^{-1} and f_p^{-1} respectively. The user can then compute its public key h as $f_q^{-1}g$. The private key consists of the polynomials f and f_p^{-1} .

[0006] When a second user wants to send the first user an encrypted message, it uses the first user's public key h . The second user also has access to the system parameters. A message m is encrypted as $e = m + prh \pmod{q}$. The value r is randomly chosen for each encryption.

[0007] Upon receipt of an encrypted message m , the recipient decrypts the message by computing $a = ef \pmod{q}$. The recipient then establishes a window in the range $\frac{-q}{2}$ to $\frac{q}{2}$. The

recipient selects coefficients for a in the window. The recipient computes $m = a f_p^{-1} \pmod{p}$.

The recipient then checks that m is in the set of valid messages. If m is in the set of valid messages, then the message has been recovered. Otherwise, the recipient chooses a new window and proceeds to select coefficients in the new window. The four mentioned steps are repeated. This may continue for multiple windows until a valid message is found. Once a valid message is found, execution will stop. If all of the possible windows are exhausted and no valid message has been found, then the recipient will experience an error condition and report that the message cannot be deciphered.

[0008] In order to avoid indecipherable messages, it has been suggested that the parameter T be chosen to be at least 30, and as large as 150. Since a large number of windows may be tested, it is likely that a valid message will be found eventually. In most cases however it is not necessary to check all of the windows.

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5 SUMMARY OF THE INVENTION

6 [0009] The inventors have recognised a vulnerability in the NTRU decryption process which
7 may be exploited to determine private keys. The vulnerability exploits indecipherable messages
8 in order to determine multiple bits of the secret key.

9 [0010] The attack proceeds by first finding one message m and one random value r such that
10 the encryption $e = m + prh \pmod{q}$ is indecipherable. This step is performed by choosing
11 random messages and values r and sending them to the victim for decryption. When decryption
12 fails, it can be noted that the message is indecipherable. In the alternative, the time required to
13 process the message may be measured. Indecipherable messages will require many windows to
14 be tested during decryption, and accordingly will require more time than valid messages. It will
15 be recognised that this attack requires that the victim decrypt messages of the attacker's
16 choosing.

17 [0011] Once the attacker has determined one particular message m and a corresponding value
18 r which yield an indecipherable encryption, the attacker then proceeds to find further
19 indecipherable messages. The attacker proceeds by choosing a new random value r_1 , and then
20 encrypting m with r_1 . The attacker then tests if $m + pr_1h$ can be decrypted. If this message
21 cannot be decrypted, then it is saved for further use in the attack. The attacker then repeats the
22 step of choosing a new random value and proceeds to find random values r_2, r_3 , etc. before
23 proceeding with the next phase of the attack.

24 [0012] Once the attacker has contained a large enough number of decipherable messages, the
25 attacker examines the r_i values which have been found. The attacker looks at each co-ordinate,
26 and counts the number of values which occur for each co-ordinate. Because of the structure of
27 the encryption equation, there is likely to be a correlation between the co-efficients of these
28 values r_i , and the co-efficients of the private polynomial g . Once the statistics have been
29 accumulated, the attacker predicts a value of g from the distribution of the co-efficients of the r_i

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values. This value of the polynomial g may then be used to determine the value of f_g^{-1} from the equation for the public key h . The attacker may thus determine all of the private values of the cryptosystem and therefore break the system

[0013] The inventors have recognised that avoiding the above attack may be performed by having the decryptor perform a constant amount of work for each decryption. This is accomplished by always testing all possible windows even when a valid message has already been found. The attacker therefore cannot determine which messages are actually indecipherable and the attack will be avoided. Preferably, the value of T is chosen to be less than 30 and more preferably less than 10 in order that the additional work from testing all windows is minimised. Ideally, the value of T will be chosen to be 1, 2, or 3.

[0014] According to one aspect there is provided a method of decrypting a message encrypted using a truncated ring cryptosystem. The method comprises selecting a window parameter T determining a plurality of windows of a predetermined size, each window being shifted by an amount less than or equal to the window parameter T . A decryption candidate is determined for each possible window. Each decryption candidate is tested to determine whether it is a valid message. The result of the decryption is chosen to be a valid message found in the previous step or if no valid message is found it is indicated that the message could not be decrypted. By this method, a constant number of decryption candidates are determined for each decryption.

[0015] According to another aspect, there is provided a method of decrypting a message encrypted using a truncated ring cryptosystem. The method comprises generating a random sequence of integers less than a fixed value, each integer corresponding to a window of a predetermined size and being shifted by the amount of the integer. Decryption candidates are successively determined for each possible window, and tested until a valid message is found, and the valid message is chosen as the result of the decryption. If no valid message is found after each possible window is used, it is indicated that the message could not be decrypted.

[0016] According to a further aspect, there is provided a method of selecting system parameters for a truncated ring cryptosystem. The method comprises selecting an initial set of parameters, generating private keys, testing the vulnerability of each private key to an attack on

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1 the cryptosystem based on determining indecipherable messages and when the cryptosystem is
2 vulnerable, repeatedly increasing the value of one of the parameters and re-testing the
3 vulnerability until the vulnerability has been reduced.

4 [0017] According to yet another aspect, there is provided a method of encryption with a
5 truncated ring cryptosystem. The method comprises using first, second and third cryptographic
6 hash functions to obtain a first string from a message and a number. The number is used as a
7 second string. The first cryptographic hash function is used to obtain a third string from the
8 message and the number. A padded message is formed from the first, second, and third strings.
9 The padded message is encrypted with an encryption function.

10 [0018] According to a yet further aspect, there is provided a truncated ring cryptographic
11 system comprising system parameters selected by testing the vulnerability of randomly chosen
12 private keys to an attack based on determining indecipherable messages, an encryption engine,
13 and a decryption engine.

14 [0019] According to still another aspect, there is provided a truncated ring cryptographic
15 system comprising system parameters including a window parameter less than 30, an encryption
16 engine, and a decryption engine.

17 [0020] According to a still further aspect, there is provided a decryptor for a truncated ring
18 cryptographic system comprising a window parameter T determining a plurality of windows of a
19 predetermined size, each window being shifted by an amount less than the window parameter T.
20 The decryptor includes a calculator to determine a decryption candidate for each possible
21 window and a tester to determine whether each decryption candidate is a valid message. A
22 selector chooses the result of the decryption to be a valid message or if no valid message is found
23 indicates that the message could not be decrypted.

24 [0021] According to yet another aspect, there is provided a decryptor for a truncated ring
25 cryptographic system comprising a random sequence of integers less than a fixed value, each
26 integer corresponding to a window of a predetermined size and being shifted by the amount of
27 the corresponding integer. The decryptor includes a calculator to determine a decryption
28 candidate for each possible window and a tester to determine whether each decryption candidate

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1 is a valid message. A selector chooses the first valid message found by the tester as the result of
2 the decryption.

3 [0022] According to still another aspect, there is provided a system parameter selector for a
4 truncated ring cryptographic system comprising an initial set of parameters, a private key
5 generator, an attack engine to determine the vulnerability of each private key to an attack on the
6 cryptosystem based on determining indecipherable messages, and a parameter updater to
7 repeatedly increase the value of one of the parameters and run the attack engine until the
8 vulnerability of the system to the attack has been reduced.

9 [0023] According to a still further aspect, there is provided an encryptor to encrypt a
10 message in a truncated ring cryptographic system comprising a first, a second, and a third
11 cryptographic hash function, and a generator to generate a number. A message padder is
12 configured to form a padded message from a first string computed using the first, second and
13 third cryptographic hash functions on the message and the number, a second string formed from
14 the number and a third string computed using the first cryptographic hash function on the
15 message and the number. An encryptor is provided to encrypt the padded message using an
16 encryption function.

17 [0024] According to one aspect there is provided a data carrier containing instructions to
18 direct a processor to decrypt a message encrypted using a truncated ring cryptosystem. The data
19 carrier includes instructions to select a window parameter T determining a plurality of windows
20 of a predetermined size, each window being shifted by an amount less than or equal to the
21 window parameter T. A decryption candidate is determined for each possible window. Each
22 decryption candidate is tested to determine whether it is a valid message. The result of the
23 decryption is chosen to be a valid message found in the previous step or if no valid message is
24 found it is indicated that the message could not be decrypted. A constant number of decryption
25 candidates are determined for each decryption.

26 [0025] According to another aspect, there is provided a data carrier containing instructions to
27 direct a processor to decrypt a message encrypted using a truncated ring cryptosystem. The data
28 carrier includes instructions to generate a random sequence of integers less than a fixed value,
29 each integer corresponding to a window of a predetermined size and being shifted by the amount

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1 of the integer. Decryption candidates are successively determined for each possible window, and
2 tested until a valid message is found, and the valid message is chosen as the result of the
3 decryption. If no valid message is found after each possible window is used, it is indicated that
4 the message could not be decrypted.

5 [0026] According to a further aspect, there is provided a data carrier containing instructions
6 to direct a processor to select system parameters for a truncated ring cryptosystem. The data
7 carrier includes instructions to select an initial set of parameters, generate private keys, test the
8 vulnerability of each private key to an attack on the cryptosystem based on determining
9 indecipherable messages and when the cryptosystem is vulnerable, repeatedly increase the value
10 of one of the parameters and re-testing the vulnerability until the vulnerability has been reduced.

11 [0027] According to yet another aspect, there is provided a data carrier containing
12 instructions to direct a processor to encrypt a message using a truncated ring cryptosystem. The
13 data carrier includes instructions to use first, second and third cryptographic hash functions to
14 obtain a first string from a message and a number. The number is used as a second string. The
15 first cryptographic hash function is used to obtain a third string from the message and the
16 number. A padded message is formed from the first, second, and third strings. The padded
17 message is encrypted with an encryption function.

18 19 BRIEF DESCRIPTION OF THE DRAWINGS

20 [0028] These and other features of the preferred embodiments of the invention will become
21 more apparent in the following detailed description in which reference is made to the appended
22 drawings wherein:

23 [0029] Figure 1 is a schematic representation of a communication system;

24 [0030] Figure 2 is a schematic representation of a method of encryption;

25 [0031] Figure 3 is a schematic representation of a method of decryption;

26 [0032] Figure 4 is a schematic representation of a method of an attack on the system of
27 Figure 1;

28 [0033] Figure 5 is a schematic representation of an alternate method of decryption;

29 [0034] Figure 6 is a schematic representation of a method of parameter selection;

- 1 [0035] Figure 7 is a schematic representation of a method of padding;
2 [0036] Figure 8 is a schematic representation of a circuit used to pad messages;
3 [0037] Figure 9 is a schematic representation of a method of padding using the circuit of
4 Figure 8;
5 [0038] Figure 10 is a schematic representation of a circuit used to recover a message from a
6 padded message; and
7 [0039] Figure 11 is a schematic representation of a method performed by the circuit of
8 Figure 10.

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10 DESCRIPTION OF THE PREFERRED EMBODIMENTS

11 [0040] Referring to Figure 1, a communication system 10 includes correspondents 12, 14
12 connected by a communication channel 16. The correspondent 12 wishes to send messages to
13 the correspondent 14, and for this purpose has access to certain public parameters of the
14 correspondent 14. The correspondent 14 has system parameters 22, private parameters 24 and a
15 public key 26. The system parameters include a degree N , two moduli p, q and a window
16 parameter T . The private parameters include randomly chosen polynomials f, g and inverses of f
17 modulo q and modulo p . The public key is a value h computed from $f_q^{-1}g$.

18 [0041] The correspondents 12, 14 also include cryptographic processors 28, 30 for
19 performing cryptographic calculations. The correspondent 12 has a copy of a system parameters
20 18 and the public 20 of correspondent 14. The correspondent 12 can therefore use these
21 parameters in order to send encrypted messages to the correspondent 14.

22 [0042] The NTRU cryptosystem as presented in US Patent No. 6, 081,597 depends on four
23 parameters (N, p, q, T) and four sets of integer polynomials of degree less than N . The sets
24 include a message space L_m , two key spaces L_f, L_g , and a nonce space L_r . All of the integer
25 polynomials belong to the ring $R = \mathbb{Z}[x]/(X^N - 1)$ and $*$ denotes multiplication in R . The
26 elements of the ring R may be represented as polynomials of a degree less than N . Operations in
27 the ring are performed by polynomial addition and multiplication with the additional identity that
28 $X^N = 1$. The parameter q is selected to be a positive integer. The parameter p can then either be a

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1 positive integer considerably smaller than q or a small polynomial (in the sense the $p(1)$ is small,
 2 in both cases p is required to be relatively prime to q in R .
 3 [0043] Polynomials in R will occasionally be reduced modulo q or p . When q and p are
 4 integers, this means reducing each coefficient modulo q or p respectively. If p is a polynomial
 5 then reducing x modulo p means finding a specific predetermined representative from the set
 6 $x + Rp = \{x + yp, y \in R\}$. Let:

$$7 \quad L(d_1, d_2) = \left\{ z \in R : \begin{cases} z \text{ has } d_1 \text{ coefficients equal to } 1, d_2 \text{ coefficients equal to } -1 \\ \text{and the remaining coefficients equal to } 0 \end{cases} \right\}.$$

8 [0044] If p is an integer then the window parameter T is usually selected to be zero and L_f ,
 9 L_g , L_r , and L_m are defined as follows. Define the message space L_m as

$$10 \quad L_m = \left\{ m \in R : m \text{ has all coefficients in } \left[\left\lfloor \frac{p-1}{2} \right\rfloor, \left\lceil \frac{p-1}{2} \right\rceil \right] \right\}$$

11 and define the key spaces L_f , L_g , and the nonce space L_r as

$$12 \quad L_f = L(d_f, d_f - 1)$$

$$13 \quad L_g = L(d_g, d_g), \text{ and}$$

$$14 \quad L_r = L(d_r, d_r),$$

15 where d_f , d_g , and d_r are positive integers, whose values depend on N , q , and p .
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17 [0045] If p is a polynomial then define L_m to be the unique representatives of the sets $x + Rp$
 18 used in reducing modulo p . Note that to aid in decryption the representatives from $x + Rp$ are
 19 selected so that the coefficients of polynomials in L_m are small. The sets L_f , L_g , L_r , and the
 20 integer T are then selected to allow the decryption algorithm to have a good probability of
 21 success. Generally this requires the coefficients of polynomials in L_f , L_g , and L_r to be small.

22 [0046] Key-pairs are generated by selecting two polynomials $f \in L_f$ and $g \in L_g$ such that
 23 there exist polynomials f_p^{-1} and f_q^{-1} satisfying

$$24 \quad f * f_p^{-1} = 1 \pmod{p} \text{ and}$$

$$25 \quad f * f_q^{-1} = 1 \pmod{q}$$

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1 The private key comprises the polynomials f and f_p^{-1} . The public key is the polynomial

2 $h = f_q^{-1}g \pmod{q}$.

3 [0047] Referring to Figure 2, a method of encrypting a message is shown by the numeral

4 100. At step 102, the correspondent 12 uses the system parameters N, p, q and T . The input

5 parameters are a message m which the correspondent 12 wishes to send to the correspondent 14

6 and the public key h of the correspondent 14. The correspondent 12 then chooses the random

7 value r at step 104. The sender then computes $e = m + prh \pmod{q}$ at step 106.

8 [0048] Upon receipt of an encrypted message e the correspondent 14 performs the steps

9 shown in Figure 3 by the numeral 200. The correspondent 14 first receives the encrypted

10 message e at step 202. It then calculates at step 204 $a \equiv ef \pmod{q}$. It then sets a window at

11 step 206. The window is initially set to the range $\frac{-q}{2}$ to $\frac{q}{2}$. Then, at step 208 the correspondent

12 14 selects coefficients of a in the current window. At step 210, the correspondent 14 computes

13 $M = a f_p^{-1} \pmod{p}$. Then, at step 212 the correspondent 14 checks that M is in the set L_m of

14 valid messages. If the message M is valid at step 214 then the correspondent 14 uses the message

15 as the recovered message at step 218. If however the message is not valid at step 214, then the

16 correspondent 14 chooses a new window at step 216 and returns to step 208 to select new

17 coefficients.

18 [0049] Given a public key h and a message $m \in L_m$, encryption E proceeds as follows.

19 Select a random element $r \in L_r$ and calculate $e = m + prh \pmod{q}$. Encryption may be denoted

20 by $E_h(m; r) = e$.

21 [0050] Given a ciphertext $e = m + prh \pmod{q}$, decryption $D_{f, f_p^{-1}}$ proceeds as follows. First

22 calculate:

$$a \equiv ef \pmod{q}$$

23 $\equiv mf + prhf \pmod{q}$

$$\equiv mf + prg \pmod{q}$$

24 [0051] Now convert the modular reduction above to an integer polynomial by choosing

25 integer representatives for the coefficients of a .

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1 [0052] The NTRU parameters were selected in such a way that, for the vast majority of m
 2 and r , all the coefficients of $mf + prg$ fall in a range of width q centred at a value that can be
 3 determined from e . (Typically the centre is the expected value of the coefficients of $mf + prg$).
 4 Thus for most m and r , a equals $mf + prg$.

5 [0053] In this case, decryption continues as follows:

$$6 \quad af_p^{-1}(\text{mod } p) \equiv (mf + prg)f_p^{-1}(\text{mod } p)$$

$$7 \quad \equiv (mf)f_p^{-1}(\text{mod } p)$$

$$8 \quad \equiv m(\text{mod } p)$$

9 By definition of the message space, if $m \in L_m$ then $m \equiv m(\text{mod } p)$ and thus a recovers the
 10 message m .

11 [0054] If the above does not recover a valid message m , the range of width q is shifted by 1
 12 and the above reduction modulo q is repeated. If this does not recover a valid message, the range
 13 is shifted by -1 and the above modular reduction repeated. In the absence of a valid message
 14 being recovered, the shifting and reduction by 2, -2 , and so on up to a shift by $-T$ at which point
 15 decryption is said to fail with a gap failure. The ciphertext in question is said to be an
 16 "indecipherable valid ciphertext."

17 [0055] Thus if $E_h^N(m; r) = e$ then $D_{f, f_p^{-1}}^N(e)$ equals m precisely when $a = ef(\text{mod } q)$ reduced
 18 to the expected range (shifted up to $\pm T$, if need be) equals $mf + prg$. This can be used to find a
 19 good characterisation for which valid ciphertext will not decipher correctly.

20 [0056] Referring to Figure 4, the method of attacking the NTRU system is shown generally
 21 by the numeral 300. The attacker first finds one message m and a nonce r such that $e = m + prh$
 22 (mod q) is indecipherable. The attacker then chooses the random value r_i at step 304. The
 23 attacker then encrypts the message m using the random value r_i at step 306. The attacker then
 24 tests if $m + pr_i h$ can be decrypted at step 308. This is performed by sending the message to the
 25 victim. It may be necessary to monitor the amount of time that the victim requires to attempt to
 26 decrypt. At step 310, the attacker saves the pair m, r_i if the result of encrypted message is
 27 indecipherable. The attacker then repeats 312, the choice of random values at step 304 and step

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1 306, 308 and 310 until it has accumulated sufficient values r_i . The attacker then examines the
 2 co-ordinates of the r_i values which have been found at step 314. The attacker then predicts the
 3 value of the private polynomial g from the distribution of the coefficients in the r_i values at step
 4 316.

5 [0057] The attack proceeds in two stages. Stage 1 : Randomly search through pairs $(m, r) \in$
 6 $L_m \times L_r$ until the ciphertext generated from the pair (m, r) is an indecipherable valid ciphertext. In
 7 practice, this is carried out by encrypting a message to another party, transmitting the ciphertext,
 8 and observing whether the ciphertext is rejected as indecipherable. Because a successful
 9 decryption with few shifts happens with high probability, it suffices to note the time interval
 10 before rejection and assume that any ciphertext not rejected almost at once is indecipherable.

11 [0058] Stage 2: Given the pair (m, r) found in Stage 1, let $y = mf$. Typically y will have one
 12 co-efficient j which is closer to the boundary of decipherability than any other co-efficient. In
 13 this case, for random \bar{r} , there will be a bias in the co-ordinates of $mf + p\bar{r}g$ that may cause the
 14 (m, r) -ciphertext to be indecipherable.

15 [0059] The attack proceeds by randomly selecting many \bar{r} and recording the value \bar{r} for
 16 which $e = m + p\bar{r}g \pmod{q}$ was indecipherable. Because of the bias in the "bad" co-ordinates of
 17 $mf + p\bar{r}g$, the values in the recorded \bar{r} will have a correlation with the secret value g . This
 18 allows g to be recovered by analysing the distributions of the values in the recorded \bar{r} . The
 19 private key can then be recovered. (First, recover the value of f from g and h ; second, determine
 20 f_p^{-1} from f .)

21 [0060] To limit the number of indecipherable (m, \bar{r}) pairs required to determine g from the
 22 \bar{r} , the attack can be combined with lattice techniques.

23 [0061] If $y = mf$ does have a large co-efficient then the rate at which the (m, \bar{r}) are
 24 indecipherable will be noticeably larger than the random m and r . Thus by analysing the rate at
 25 which \bar{r} are found, we can determine whether y has no large co-efficients, at which point we can
 26 simply return to Stage 1. If $y = mf$ has two or more large co-efficients which are equally close to
 27 the decipherable boundary then the distributions of values in the \bar{r} may not reveal g and the
 28 attack will need to return to Stage 1. Note that in this latter case, some information about g may

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1 still be determined. In practice, for randomly determined indecipherable $(m, r,)$ there is a good
2 chance that $y = mf$ has the desired properties. Thus we expect that the need to loop to Stage 1
3 will be infrequent.

4 [0062] Referring to Figure 5, an embodiment of the invention in which alternate decryption
5 is used is shown generally by the numeral 400. The recipient first receives an encrypted message
6 e at step 402. It then calculates $a \equiv ef \pmod{q}$ at step 404. The recipient then selects co-
7 efficients in the current window at step 406. At step 408 the recipient computes $M = af_p^{-1} \pmod{p}$.
8 The recipient then checks if M is in the set of valid messages at step 410. If the message is
9 valid at step 412, then the recipient records the valid message at step 414. It then proceeds to
10 repeat the steps for each window at step 416. If the message is not valid then the recipient also
11 repeats the steps for each window at step 416. In this way, the recipient performs the same
12 number of operations regardless of how soon it finds a valid message.

13 [0063] To protect against timing attacks, such as the above, it will be recognised that the
14 decryption algorithm has been modified so that a constant amount of work is always done per
15 ciphertext. This is accomplished by proceeding with the decryption steps for each of the $2T + 1$
16 possible reduction ranges for a regardless of whether or not the message has been recovered.

17 [0064] In an alternative embodiment, randomness is introduced into the sequence of
18 windows. The possible windows $1, -1, 2, -2, \dots, -T, -T$ are randomly rearranged. This selection
19 of windows will reduce the information revealed by an indecipherable ciphertext since the
20 attacker will not know which windows have been tried and in which order.

21 [0065] In another embodiment of the invention, shown in Figure 6 by the numeral 500, the
22 system parameters are chosen in order to reduce the likelihood of finding an indecipherable
23 message. The likelihood of finding indecipherable messages is related to the system parameters
24 $N, p, q,$ and T . Values for these parameters are initially chosen 502 to set up the cryptosystem.
25 Once a private key and public key are generated, 504, the likelihood of finding indecipherable
26 messages is calculated 506. If this value is more than a predetermined value 508, then one of the
27 system parameters is modified 510. If not, then the parameters are used 512. The process may
28 be repeated until desirable parameters are found. Preferably, q is increased in order to expand
29 the window for co-efficients modulo q .

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1 [0066] In an alternative embodiment, the decryptor monitors received encrypted messages,
 2 When a large number of indecipherable messages are detected, the decryptor selects new system
 3 parameters. Preferably, the new parameters provide a lower likelihood of obtaining
 4 indecipherable messages.

5 [0067] In another embodiment, messages m (602) are padded with the nonce (604) as shown
 6 in Figure 7 in order to provide randomness throughout the message. This may be done by
 7 splitting the message into two parts m_1, m_2 and the nonce into two parts r_1 , and r_2 . Then the
 8 encrypted operation is performed on the concatenation $m_1 \parallel r_1 \parallel m_2 \parallel r_2$ (606, 608, 610, 617).
 9 Additional parts may be used to further mix bits of m with bits of r .

10 [0068] In a further embodiment shown in Figures 8 and 9, an alternative method of padding
 11 messages is used. Referring to Figure 8, a circuit is shown generally by the numeral 700. The
 12 circuit 700 includes registers 702 and 704 which hold a message M and a random string R ,
 13 respectively. The number of bits in the message m is denoted by k_1 , and the number of bits in the
 14 random string R is denoted by k_2 . The circuit 700 outputs a padded message m and a padded
 15 nonce r of bit lengths m_{len} and r_{len} respectively. The length m_{len} is at least $k_1 + k_2$. The circuit
 16 uses a hash function F 706, a hash function G 714 and a hash function H 708. The hash
 17 functions F and H take as input a binary string of length $k_1 + k_2$. The output of F is $k_3 = m_{len} - k_1 -$
 18 k_2 bits, hash function G takes input of $k_2 + k_3$ bits and produces output of k_1 bits. The hash
 19 functions F and H are connected to a concatenation of registers 702 and 704 to receive input of
 20 the binary string $M \parallel R$ of length $k_1 + k_2$. The output of the hash function F 706 is a value
 21 $a = F(M \parallel R)$ 710. The hash function G 714 operates on a concatenation of R 704 and a 710 of bit
 22 length k_2 . The output of the hash function G 714 is a value $G(R \parallel a)$ 716 of k_1 bits. The register
 23 716 is connected to an XOR gate 718. The register M 702 is also connected to the XOR gate
 24 718. The output of the XOR gate 718 is a register 720 containing $bM + G(R \parallel a)$ of k_1 bits. The
 25 resulting message m is a concatenation of registers 720, 704, and 710 of $k_1 + k_2 + k_3$ bits.
 26 Notationally, $m = b \parallel R \parallel a$. The output of the hash function H 708 is a value $d = H(M \parallel R)$ 712 of
 27 r_{len} bits. The value of d is used as the nonce r .

28 [0069] In operation of the circuit 700, the steps 800 to Figure 9 are performed. First, at step
 29 802, a message M is obtained. Then, a random string R is obtained at step 804. Then the value a

1 is computed at step 806. The value a is equal to the value of the hash function F applied to $M ||$
2 R , the concatenation of M and R . The value b is then computed at step 808 as $M \oplus G(R || a)$.
3 At step 810, the values c and d are computed, where $c = b || R || a$ and $d = H(M || R)$. Finally, the
4 result in values m and r are encrypted at step 812. Once the values have been encrypted as
5 shown in Figure 9, a recipient will be able to decrypt them and obtain the original message M .
6 Because of the padding, the additional steps of Figures 10 and 11 will be used by the recipient.
7 [0070] Referring therefore to Figure 10, a circuit for recovering the message from the padded
8 message is shown generally by the numeral 900. The circuit 900 takes as input an encrypted
9 message 902. The circuit then applies the NTRU decryption method 904. The result of the
10 decryption is a padded message m and a padded nonce r in registers 906, 908 respectively.
11 Assuming the decryption is successful, these values will be equal to the values encrypted by the
12 sender. The circuit 900 then splits the register 906 into three values b , R , a in registers 910, 912
13 and 914 respectively. The hash function G 714 is connected to the registers 912 and 914 to
14 produce a value $G(R || a)$ stored in register 920. An XOR gate 918 is connected to the values b ,
15 and register 910 and the register 920. The XOR gate produces a value in register 922 which is
16 equal to $b \oplus G(R || a)$. The hash function F 706 is connected to the registers 922 and 912 to
17 produce a value of $F(M || R)$ in register 924. A comparator 926 operates to compare register 924
18 to the value a in register 914.
19 [0071] Referring to Figure 11, the steps performed by the circuit 900 are shown generally by
20 the numeral 1000. An encrypted message is first obtained at step 1002. Then the encrypted
21 message is decrypted with NTRU at step 1004. At step 1006 the value m is split into its
22 components b , R and a . Then the value $M = b \oplus G(R || a)$ is computed at step 1008. At step 1010,
23 the value $F(m || R)$ is compared to the value a . If the values are not the same, then the message
24 is reported as invalid and step 1012, otherwise, these values are equal and the messages reported
25 as valid at step 1014. The value M is then returned at step 1016 as the result of the decryption.
26 [0072] Now consider the control over m and r of an attacker who can select M and R . Since
27 r is generated from $d = H(M || R)$ the attacker has no direct control over any of the bits which
28 determine r . The attacker does have control over R and thus since m is generated from $c = b || R || a$
29 the attacker can control at least k_2 of the m len bits used to form m . Since $a = F(M || R)$ and

1 $b = M \oplus G(R \parallel a)$ the attacker does not have direct control over any of the bits of a or b (The
2 bits of a and b will change randomly when any of the bits of R or M change). This said, the
3 attacker can exert some control over the bits of a , b and d repeatedly trying M , R combinations.
4 However, as a , b , and d will change randomly for each M and R this control is limited by the
5 amount of work which an adversary can perform.

6 [0062] It is recognized that there are many variations of this padding scheme which provide
7 the desired features. These include permuting the orders of the bit strings concatenated to form
8 a , b , c , or d , as well as replacing b with $M \oplus G'(a)$ and d with $H(M \oplus G(a))$, $H(M \parallel a)$ or H
9 $(R \parallel a)$, (where G' and H are hash functions of the appropriate lengths.

10 [0073] Although the invention has been described with reference to certain specific
11 embodiments, various modifications thereof will be apparent to those skilled in the art without
12 departing from the spirit and scope of the invention as outlined in the claims appended hereto.